Model-versioning-in-the-large: Algebraic foundations and the tile notation

Work in progress

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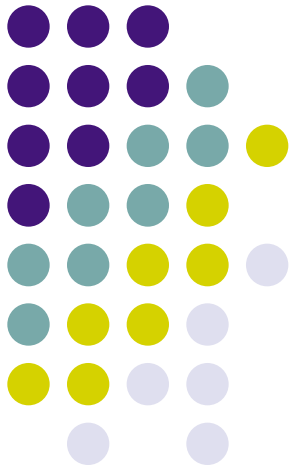
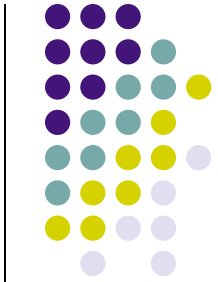


A “very large” picture:

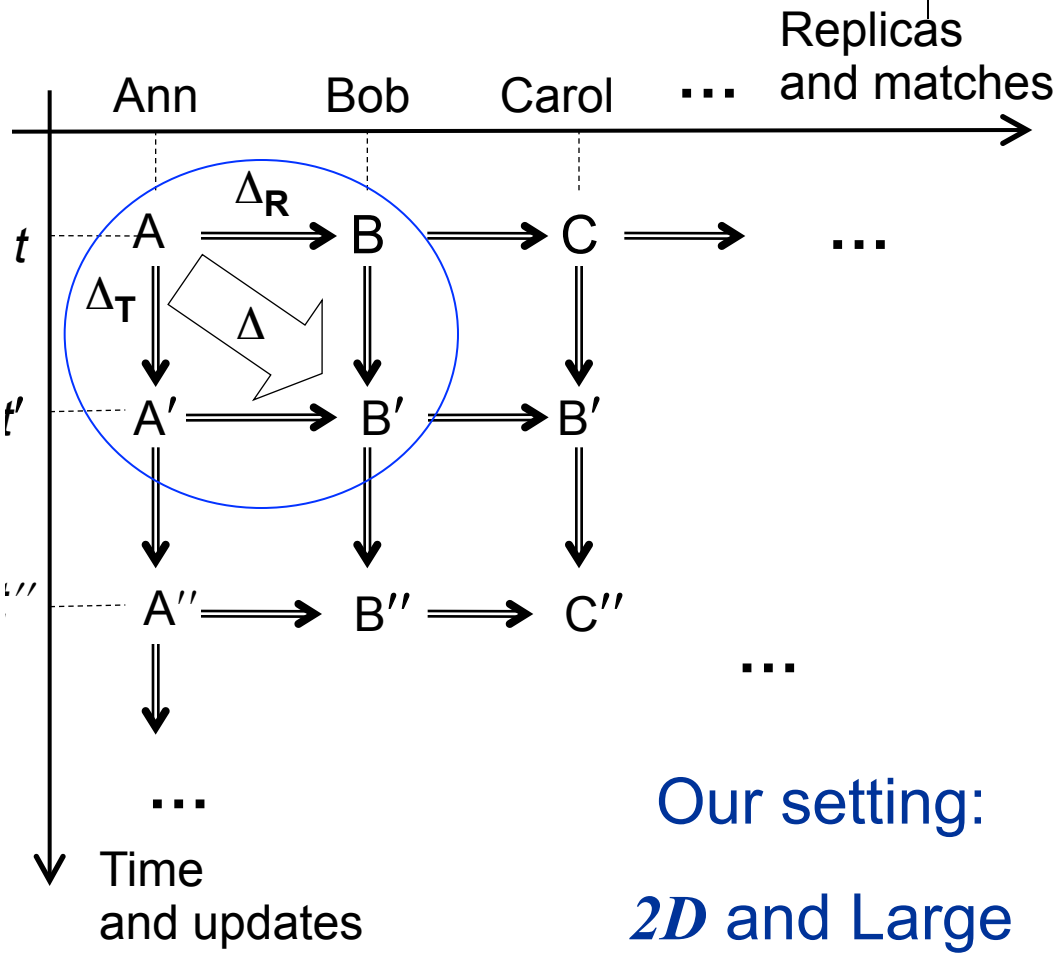


- Pictures/diagrams are good but informal (no formal semantics)
- Formulas are bad (mind boggling) but precise
- The best of the two worlds: precise diagrams with formal semantics

Versioning: small vs. large

Plan of the talk:

1. Inside a tile
2. Tile composition
3. Reconciliation
4. Sample “large” scenario
5. Summary/discussion



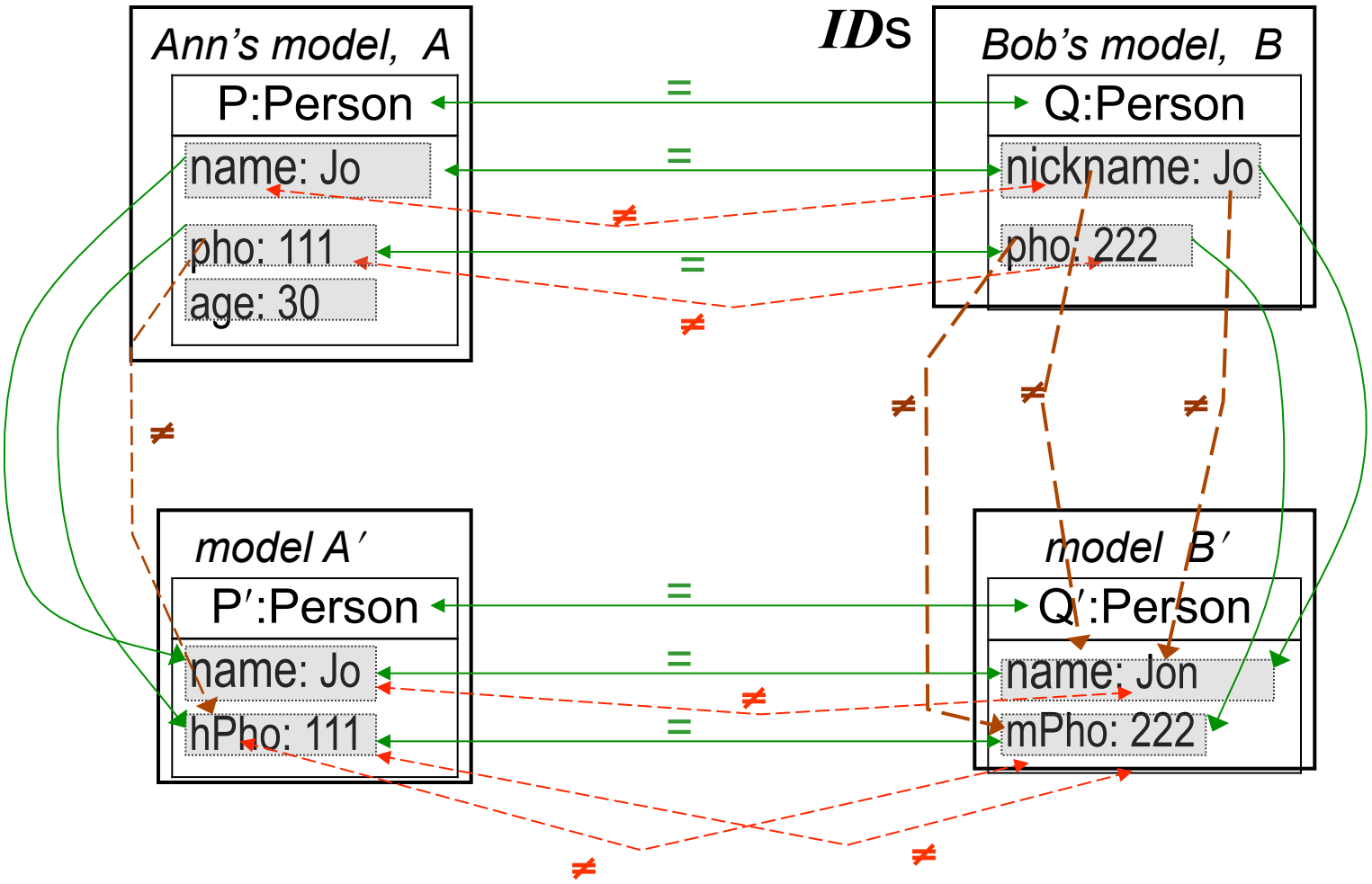
Our setting:

2D and Large

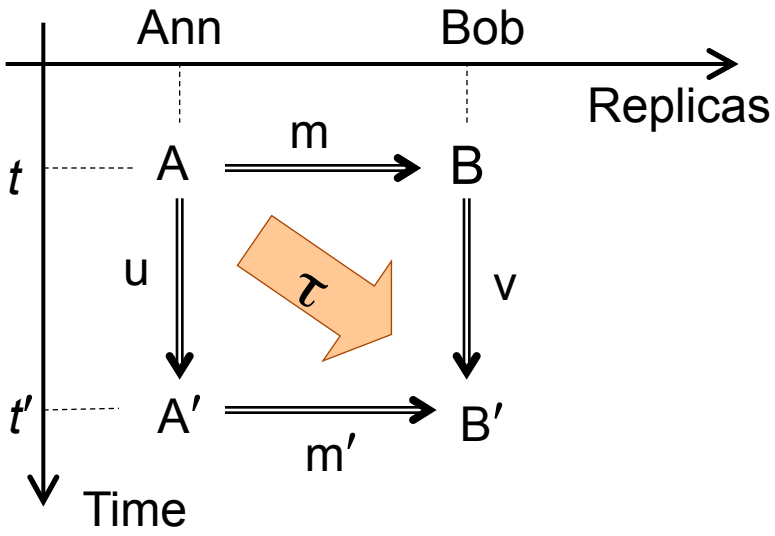
Example: Relational tile

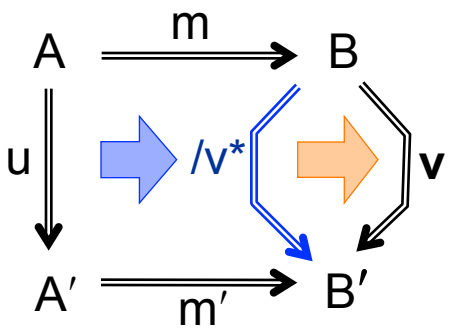
Green refers to heuristic activity;

Frame denotes elements' *ID*s

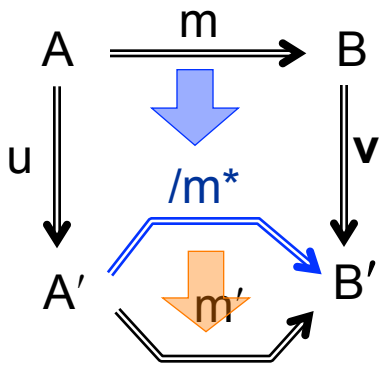


Versioning-in-the-small: Deltas are *tiles*





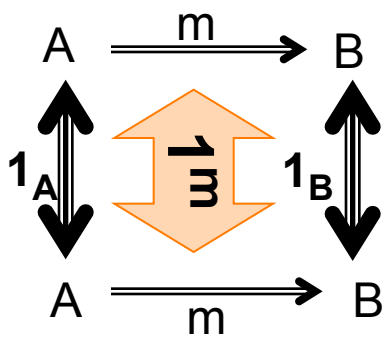
A tile as a revision of update



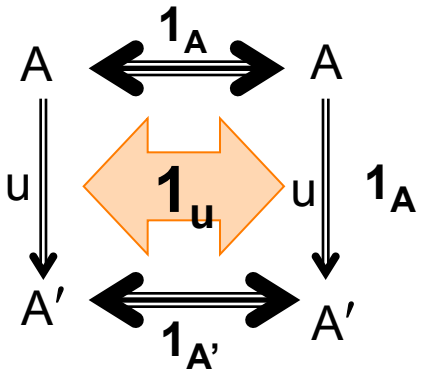
A tile as a revision of match

Blue elements are derived (computed)

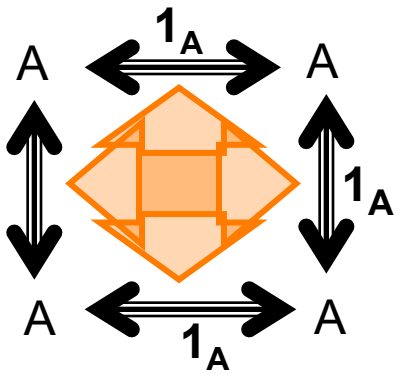
Special cases: Idle/identity tiles



(a1) Vertical identity

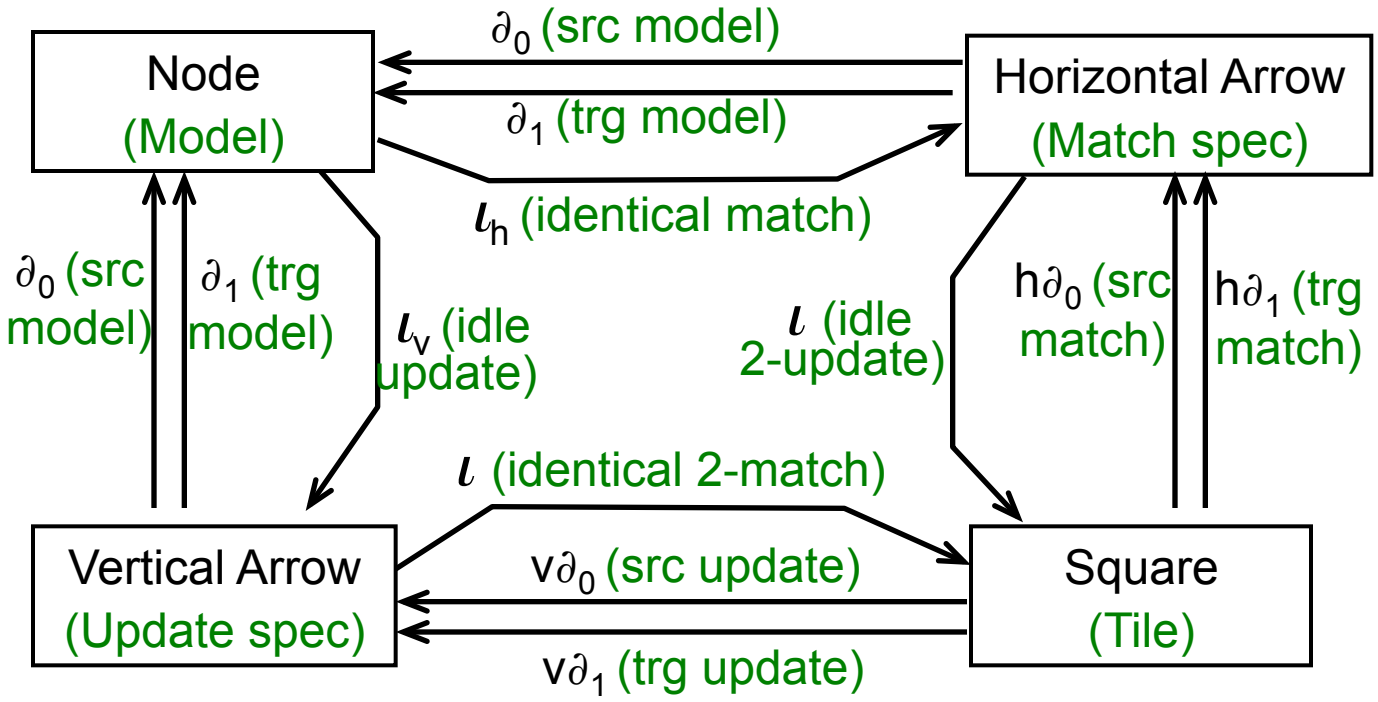


(b1) Horizontal identity

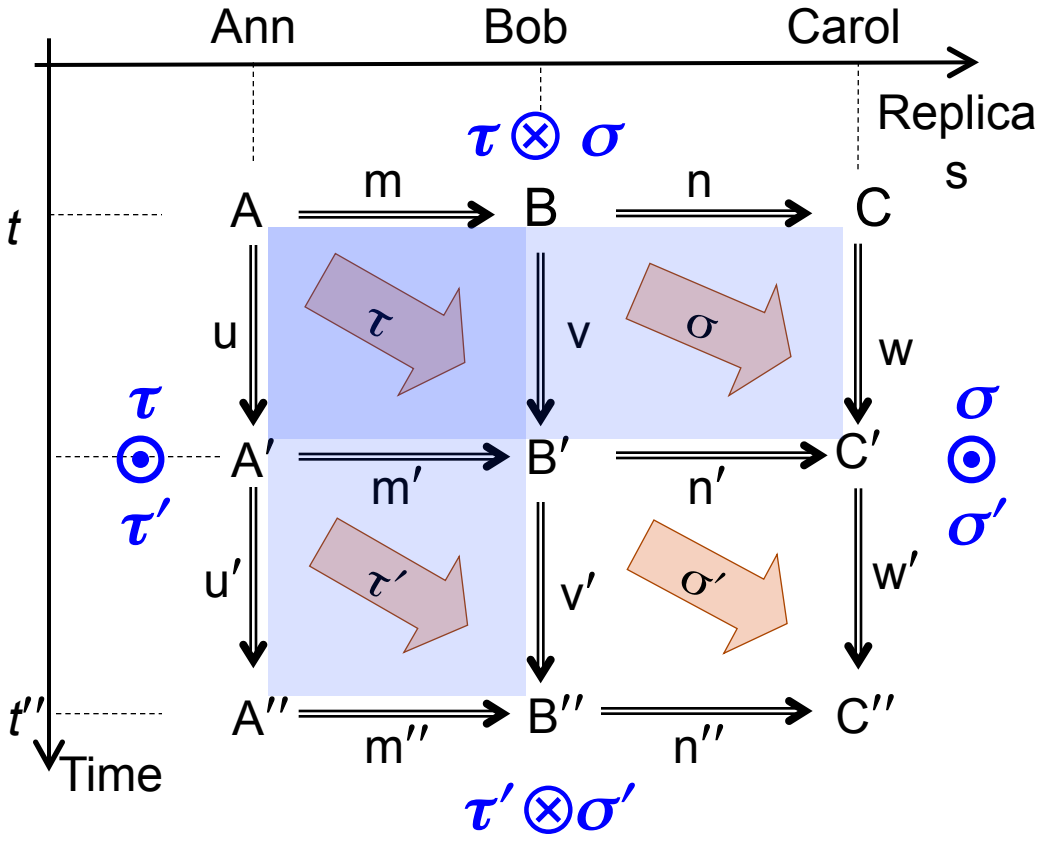


(ab) $1^{1A} = 1_{1A}$

Metamodel of tiles



Model versioning-in-the-large. Tile composition & interchange law



$$AC''_1 = (\tau \odot \tau') \otimes (\sigma \odot \sigma')$$

$$AC''_2 = (\tau \otimes \sigma) \odot (\tau' \otimes \sigma')$$

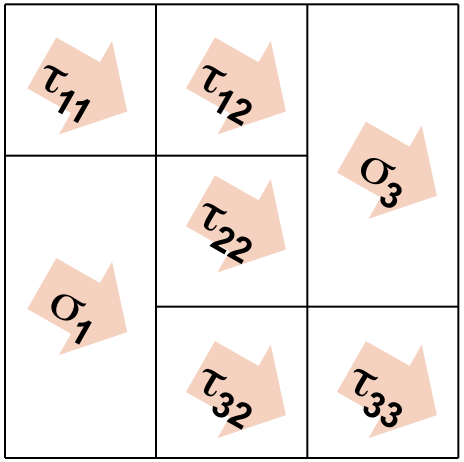
Interchange law:

$$AC''_1 = AC''_2$$

Model versioning-in-the-large. Definition of *tile system* (double category)

- Collection of nodes, vertical and horizontal arrows, and squares (tiles)
- V-arrows can be composed (assoc. and units) and h-arrows can be composed (assoc. and units).
- Tiles can be composed vertically (assoc. and units) and horizontally (assoc. and units), and work together under the interchange law.

Model versioning-in-the-large. Pasting lemma: Any tile system has the following property:

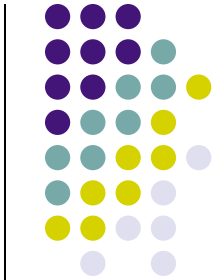


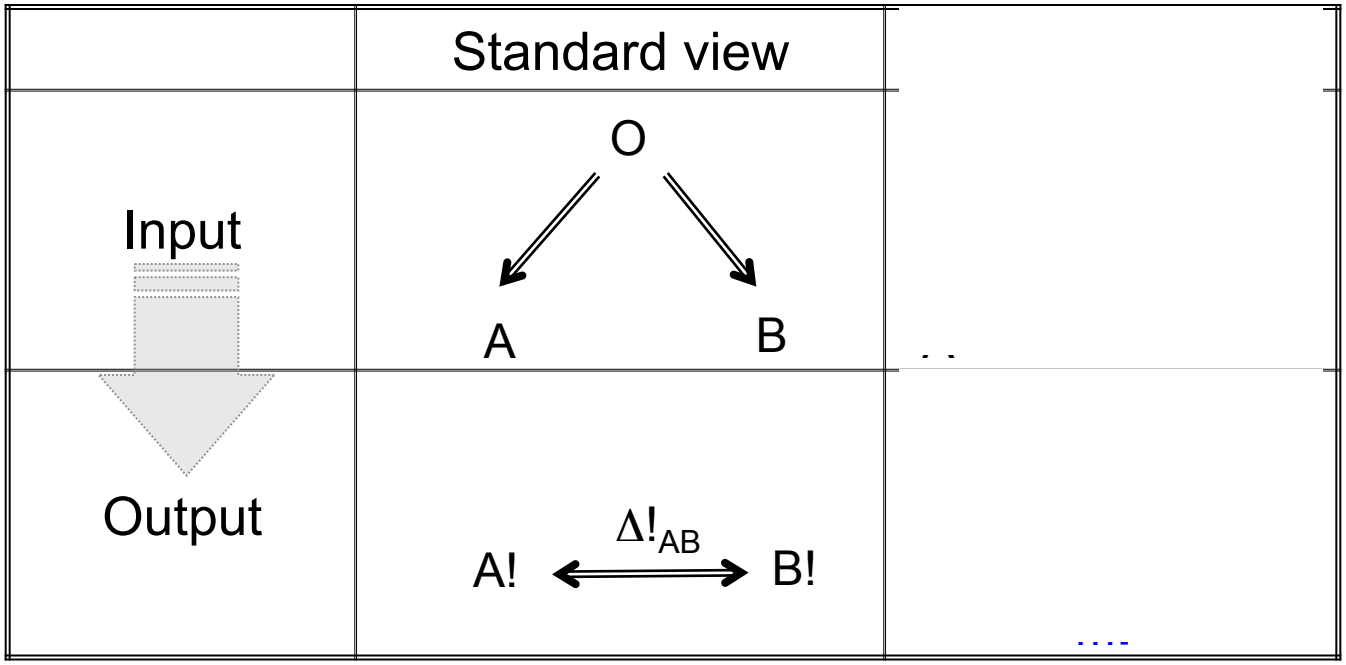
$$\{(\tau_{11}\otimes\tau_{12})[\sigma_1\otimes(\tau_{22}\,\tau_{32})]\}\otimes(\sigma_3\tau_{33}) =$$

$$(\tau_{11}\sigma_1)\otimes(\tau_{12}\tau_{22}\tau_{32})\otimes(\sigma_3\tau_{33}) \quad =$$

$$(\tau_{11}\sigma_1)\otimes\{[(\tau_{12}\tau_{22})\otimes\sigma_3](\tau_{32}\otimes\tau_{33})\}$$

Optimistic versioning and reconciliation



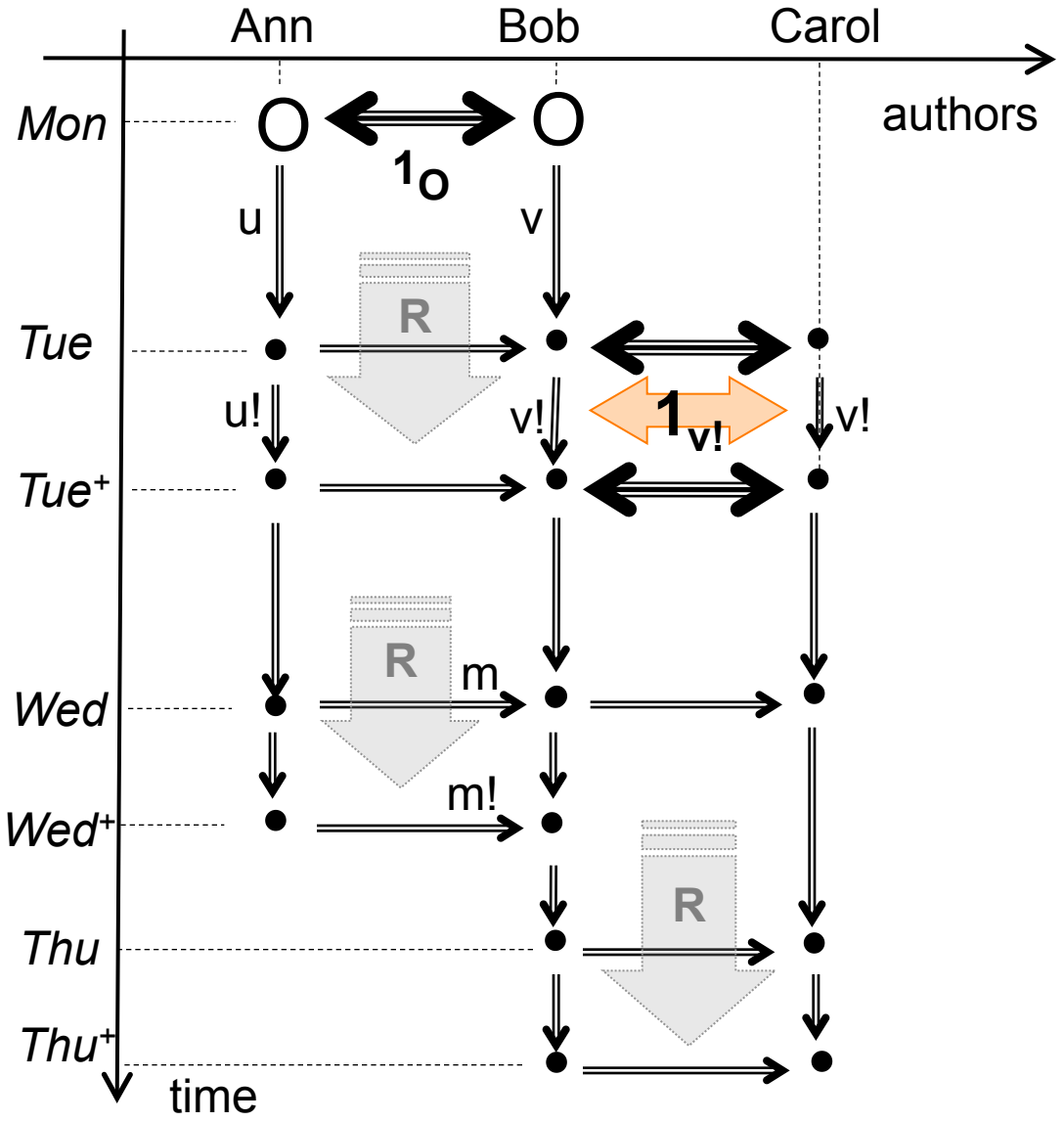


Algebraic laws:

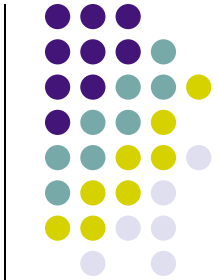
(1) $(\tau \otimes \sigma)! = \tau! \otimes \sigma!$

(2) $(\tau \odot \tau')! = \tau'!$ (optional)

Use case: “large” versioning scenario via tiles



Summary



- The elementary unit (molecule) of model versioning is a 2D-structure -- tile. Complex scenarios are composed from tiles.
- Tile composition is regulated by algebraic laws of double categories (associativity, interchange law, pasting lemma).
- Complex scenarios are terms built from tiles in some signature of tile operations. Hence,
- Algebraic machineries of category theory become applicable (diagram chasing/diagrammatic calculus).

Even bigger picture

- Engineering (e.g., mechanical and electrical)
- Physics
- Mathematics
- Category theory (abstract nonsense)
  - Higher-dimensional category theory

Software Engineering

Thank you!

Questions/Comments?